Electrical Resistivity of Metals at Low Temperatures

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For the system of the coupled Boltzmann transport equations for electrons, phonons and impurities in metals the influence of the crosscoupling terms is investigated. It is shown that this results at low temperatures in a contribution to the electrical resistivity proportional to $1/\sqrt{T}$. This is due to the fact that by phonon-impurity scattering the phonon-drag contribution is influenced by the presence of the impurities. A comparison with recently published experimental work is made.

In the preceding paper [1] (referred to as I in the following) the coupled system of Boltzmann transport equations for electrons, phonons and impurities in metals was solved for high temperatures. It was shown that on account of the cross-coupling terms in the transport equations the resistance by impurities varies with temperature.

Carrying through the same calculations for low temperatures and an external electrical field E_x, we are led to two equations for the deviations $\Gamma(\varepsilon)$ and $\Psi(u)$ from equilibrium for electrons and phonons, respectively. These two equations are (using the same symbols as in I):

$$\Psi(u) = \Gamma(1) \frac{u^2(1-u)}{2\hbar c_1 q_D (u^3(1-u) + a/\sqrt{T}\, u^5/N(u,\beta))} \tag{1}$$

with

$$a = \frac{M^{1/2} P n_i \hbar^2}{3(2\pi)^{3/2} D m^2 \sqrt{k_B}\, c_1 q_D} \tag{2}$$

and

$$e E_x = \Gamma(\varepsilon) \left[\frac{\sqrt{m}\, D\, \Omega\, q_D^4}{2^{7/2} \pi \varepsilon_F^{3/2} \hbar} J_5^{\text{Debye}} \left(\frac{T}{\theta}\right)^5 \varepsilon^{-3/2} + \frac{\pi \sigma_{ei} n_i}{2^{3/2} \hbar} \sqrt{m \varepsilon_F}\, \sqrt{\varepsilon} + \frac{m}{\hbar \tau_{ee}} \right] + \Gamma(1) \frac{\sqrt{m}\, D\, \Omega\, q_D^4}{2^{7/2} \pi \varepsilon_F^{3/2} \hbar} I(a/\sqrt{T}) \tag{3}$$

with

$$I(a/\sqrt{T}) = \int_0^1 \frac{u^7 (1-u)^2}{u^3(1-u) + a T^{-0.5} u^5/N(u,\beta)} \, du\,. \tag{4}$$

Contrary to I we now have to take care of the electron-electron-interaction, which we simply include by a relaxation time $\tau_{ee}^{-1} \propto T^2$. Furthermore we neglect the phonon-phonon-interaction altogether.

The electrical current is defined by

$$j_e = \Gamma(1) \frac{n_e e}{\hbar} = \frac{1}{\varrho} E_x\,, \tag{5}$$

and we obtain for the resistivity:

$$\varrho = \frac{\sqrt{m}\, D\, \Omega\, q_D^4}{2^{7/2} \pi \varepsilon_F^{3/2} n_e e^2} \left[J_5^D \left(\frac{T}{\theta}\right)^5 + I(a/\sqrt{T}) \right] + \frac{m}{n_e e^2 \tau_{ee}} + \frac{\pi \sigma_{ei} n_i}{2^{3/2} n_e e^2} \sqrt{m \varepsilon_F}\,; \tag{6}$$

J_5^D is a be bye integral, its value at low temperatures is 124.4.

When no impurities are present at all, the quantity $I(a/\sqrt{T})$ is constant. For small a and not too low temperatures T we may expand $I(a/\sqrt{T})$ in ascending powers of $a/\sqrt{T}$ and obtain in first order

$$I(a/\sqrt{T}) = \frac{1}{30} - \frac{a I_1(\beta)}{\sqrt{T}} \pm \dots \tag{7}$$

(for $I_1(\beta)$ see paper I).

Neglecting the phonon contribution with T^5, we are led to an approximate formula for the resistivity of metals at low temperatures:

$$\varrho = \varrho_0 + A T^2 + B/\sqrt{T}\,, \tag{8}$$

$$B = -\frac{\Omega M^{1/2} P n_h n_i I_1(\beta)}{3(2\pi)^{5/2} \hbar c_1 \sqrt{k_B}\, e^2 n_e^2}\,. \tag{9}$$

It should be noted that the additional contribution $B/\sqrt{T}$ is proportional to the concentration of the impurities. The resistivity given by (6) increases monotonously with temperature and does not exhibit a minimum.

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Apparently the additional term $B/\sqrt{T}$ has been observed by Lee et al. [2], as we have checked by a preliminary evaluation of their data for their sample K 6. An anomalous behavior of the resistivity of alloys at low temperatures has been predicted by Altshuler and Fleurov et al. [3], originating from an entirely different mechanism. They found a minimum of the resistivity and their additional term varies with $\sqrt{T}$ and $\varrho_0^{5/2}$. This has been observed by Warnecke and Gey [4] for the system germanium in copper at a concentration of about 10 atom percent, whereas Lee et al. used very pure potassium. It is obvious that the Altshuler-Aronov term supercedes the contribution $B/\sqrt{T}$ at high concentrations of impurities.

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